 Name: ____________________

Vector Addition by Components

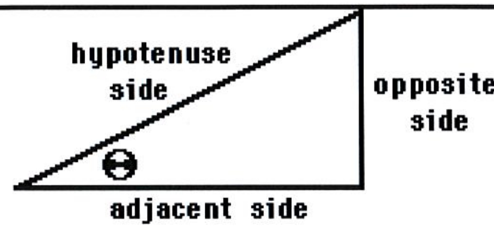
Read from **Lesson 1** of the **Vectors and Motion in Two-Dimensions chapter** at **The Physics Classroom:**
http://www.physicsclassroom.com/Class/vectors/u3l1eb.cfm

MOP Connection: Vectors and Projectiles: sublevels 3 and 4

TIP
Trigonometry Review

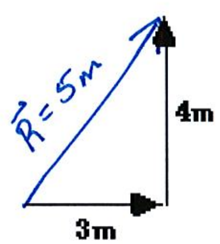
Trigonometric functions are mathematical functions that relate the length of the sides of a right triangle to the angles of the triangle. The meaning of the functions can be easily remembered by the mnemonic

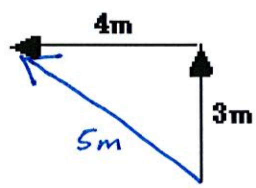
SOH CAH TOA

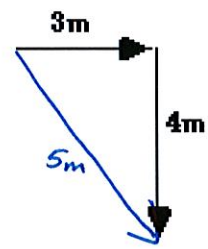


SOH --> Sin Θ = $\frac{\text{Opposite}}{\text{Hypoteneuse}}$ **CAH** --> Cos Θ = $\frac{\text{Adjacent}}{\text{Hypoteneuse}}$ **TOA** --> Tan Θ = $\frac{\text{Opposite}}{\text{Adjacent}}$

1. For the following vector addition diagrams, use Pythagorean Theorem to determine the magnitude of the resultant. Use SOH CAH TOA to determine the direction. **PSAYW**





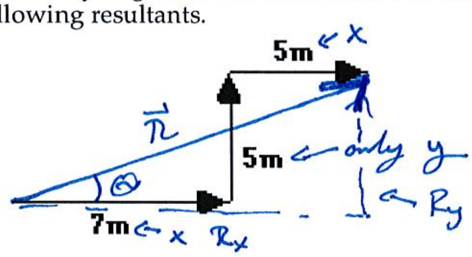


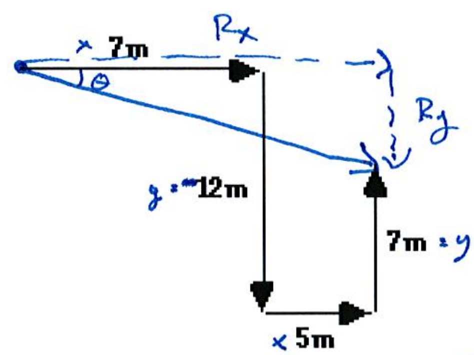
$R = \sqrt{(3)^2 + (4)^2}$

$R = 5m$

same work

2. Use the Pythagorean Theorem and SOH CAH TOA to determine the magnitude and direction of the following resultants.





$R_x = 7m + 5m = 12m$

$R_y = 5m$

$\tan\Theta = \frac{R_y}{R_x}$

$\vec{R} = \sqrt{(12)^2 + (5)^2} = \boxed{13m}$

$\Theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$

$\boxed{\Theta = 22.6°}$

$\vec{R} = 13m @ 22.6°$ N of E

$R_x = 7m + 5m = 12m$

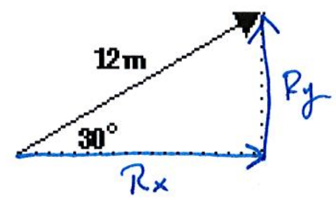
$R_y = -12m + 7m = -5m$

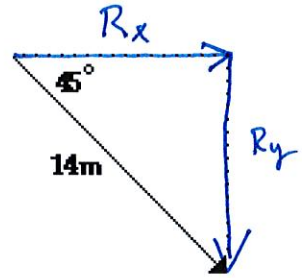
$R = \sqrt{(12)^2 + (5)^2} = 13m$

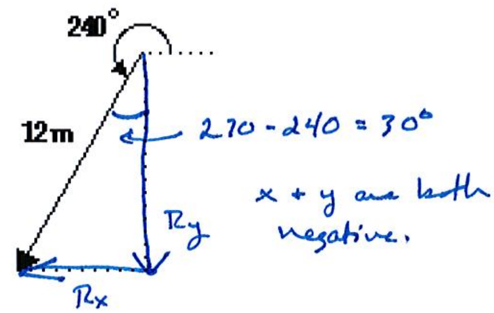
$\Theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = 22.6°$

$\vec{R} = 13m @ 22.6°$ S of E

3. A component is the effect of a vector in a given x- or y- direction. A component can be thought of as the projection of a vector onto the nearest x- or y-axis. SOH CAH TOA allows a student to determine a component from the magnitude and direction of a vector. Determine the components of the following vectors.







$\cos\theta = \frac{R_x}{12m}$

$R_x = 12m \cos(30°)$

$R_x = 10.4m$

$\sin\theta = R_y/12m$

$R_y = 12m \sin 30°$

$R_y = 6m$

$R_x = 14m \cos 45°$

$R_x = 9.9m$

$R_y = 14m \sin(45°)$

$R_y = 9.9m$

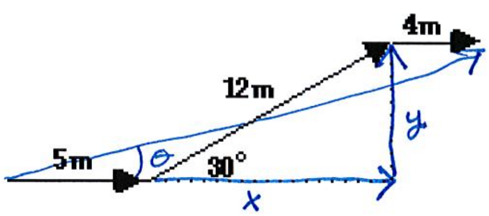
$R_x = 12m \sin(30°)$

$R_x = -6m$

$R_y = 12m \cos(30°)$

$R_y = -10.4m$

4. Consider the following vector diagrams for the displacement of a hiker. For any *angled* vector, use SOH CAH TOA to determine the components. Then sketch the resultant and determine the magnitude and direction of the resultant.



$x = 12m \cos 30°$

$x = 10.4m$

$y = 12m \sin 30°$

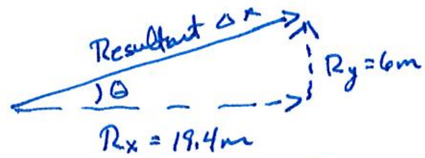
$y = 6m$

Total x movement

$R_x = 5m + 10.4m + 4m = 19.4m$

Total y movement

$R_y = 6m$

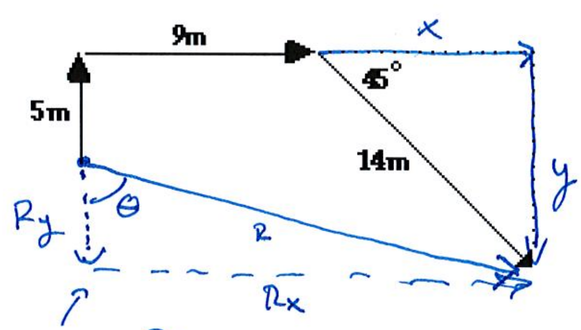


$\vec{R} = \sqrt{(19.4)^2 + (6)^2} = 20.3m$

$\tan\theta = \frac{R_y}{R_x}$ $\theta = \tan^{-1}\left(\frac{6m}{19.4m}\right)$

$\theta = 17.2°$

$R = 20.3m @ 17.2°$



Total x movement

$R_x = 9 + 14\cos 45° = 18.9m$

Total y movement

$R_y = 5m + (-14\sin(45°)) = -4.9m$

$\vec{R} = \sqrt{(18.9)^2 + (4.9)^2} = 19.5m$

$\tan\theta = \frac{R_x}{R_y}$

$\theta = \tan^{-1}\left(\frac{R_x}{R_y}\right) = 75.5°$

$\vec{R} = 19.5m @ 75.5°$ E of S

or

$\vec{R} = 19.5m @ 14.5°$ S of E

Relative Velocity and Riverboat Problems

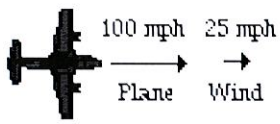
Read from **Lesson 1** of the **Vectors and Motion in Two-Dimensions chapter** at **The Physics Classroom**:

http://www.physicsclassroom.com/Class/vectors/u3l1f.html
http://www.physicsclassroom.com/Class/vectors/u3l1g.html

MOP Connection: Vectors and Projectiles: sublevel 6 (and maybe sublevel 5)

1. Planes fly in a medium of moving air (winds), providing an example of relative motion. If the speedometer reads 100 mi/hr, then the plane moves 100 mi/hr relative to the air. But since the air is moving, the plane's speed relative to the ground will be different than 100 mi/hr. Suppose a plane with a 100 mi/hr air speed encounters a tail wind, a head wind and a side wind. Determine the resulting velocity (magnitude and CCW direction) of the plane for each situation.

Tail Wind



Magnitude:

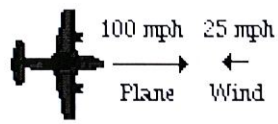
$V_{RG} = V_{PA} + V_{AG}$

$V_{PG} = 100 + 25 = 125$ mph

CCW Direction:

at 0° due East

Head Wind



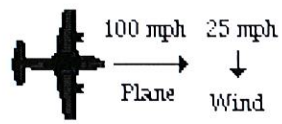
Magnitude:

$100 - 25 = 75$ mph

CCW Direction:

0° due East

Side Wind



Magnitude:

100 mph
θ
R
25 mph

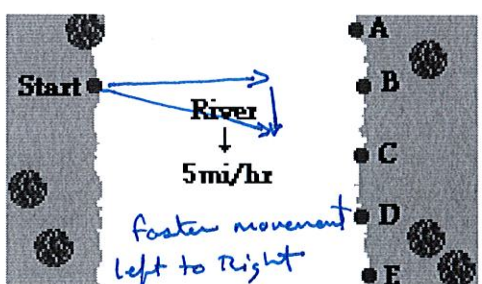
CCW Direction:

$R = \sqrt{(100)^2 + (25)^2} = 103.1$ mph

$\theta = \tan^{-1}\left(\frac{25}{100}\right) = 14°$ south of East

2. The situation of a plane moving in the medium of moving air is similar to a motorboat moving in the medium of moving water. In a river, a boat moves relative to the water and the water moves relative to the shore. The result is that the resultant velocity of the boat is different than the boat's speedometer reading, *thanks to* the movement of the water that the boat is in. In the diagram below, a top view of a river is shown. A boat starts on the west side (left side) of the river and heads a variety of directions to get to the other side. The river flows south (down). Match the boat headings and boat speeds to the indicated destinations. Use each letter once.

one example



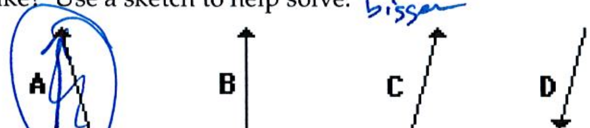
faster movement left to Right
less distance traveled N→S

Boat Heading	Boat Speed	Destination (A, B, C, D or E)	
→	14 mi/hr	D	below B
→	7 mi/hr	E	below B
↗	7 mi/hr	B	
→	20 mi/hr	C	below B
↗	12 mi/hr	A	

Total
heading
wind

3. A pilot wishes to fly due North from the Benthere Airport to the Donthat Airport. The wind is blowing *out of the Southwest* at 30 mi/hr. The small plane averages a velocity of 180 mi/hr. What heading should the pilot take? Use a sketch to help solve.

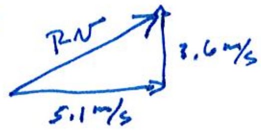
bigger



path
wind
W
N
Desired path
E
wind
S

4. A riverboat heads east on a river that flows north. The riverboat is moving at 5.1 m/s with respect to the water. The water moves north with respect to the shore at a speed of 3.6 m/s.
   a. Determine the resultant velocity of the riverboat (velocity with respect to the shore).

~~9.1 m/s~~



$V_{RS} = \sqrt{(3.6)^2 + (5.1)^2}$

$\boxed{V_{RS} = 6.2 \text{ m/s}}$

   b. If the river is 71.0 m wide, then determine the time required for the boat to cross the river.

x direction so only x component of velocity matters

$V_{RS(x)} = 5.1 \text{ m/s}$ $\quad x = vt \quad t = \frac{71\text{m}}{5.1\text{ m/s}} = 13.9\text{ s}$

   c. Determine the distance that the boat will travel downstream.

only effected by y component or 3.6 m/s

time is same. $\boxed{y} = vt = 3.6\text{ m/s}\,(13.9\text{s}) = \boxed{50\text{m}}$

5. Suppose that the boat attempts this same task of crossing the river (5.1 m/s with respect to the water) on a day in which the river current is greater, moving at 4.7 m/s with respect to the shore. Determine the same three quantities - (a) resultant velocity, (b) time to cross the river, and (c) distance downstream.

same steps (a) $V_{RS} = 6.9$ m/s

(b) $t = 13.9$ s

(c) $y = 65.3$ m

6. For a boat *heading* straight across a river, does the speed at which the river flows effect the time required for the boat to cross the river? <u>NO</u> Explain your answer.

only the East west velocity of the boat matters. The current doesn't increase the speed East to west at all since it is perpendicular to motion.

7. Repeat the same three riverboat calculations for the following two sets of given quantities.

Velocity of boat (w.r.t. water) = 3.2 m/s, East Velocity of river (w.r.t. shore) = 4.4 m/s, South Width of river = 127 m	Velocity of boat (w.r.t. water) = 2.6 m/s, West Velocity of river (w.r.t. shore) = 4.2 m/s, South Width of river = 96 m
a. Resultant velocity: magnitude = 5.4 m/s direction = 53° S of East	a. Resultant velocity: magnitude = 4.9 m/s direction = 58° S of W
b. Time to cross river = 39.7 s	b. Time to cross river = 36.9 s
c. Distance downstream = 174.7 m	c. Distance downstream = 154 m